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CONSIDERATIONS ON UNEVOLVENT PROFILES SCRAPING

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ABSTRACT: non - evolventic profiles are one of the most tricky issues related to mechanical processes. Because of the shape of the gear's profile to be processed, the processing tool must conjuge so that the wheel can be processed in accordance to the appropriate design.

Processing a gear with a grinding disc, the tooth of which has a certain profile , is an issue of extreme importance in terms of theoretical and practical. The use of unevolventic profiles in an increasingly wide range of gearings, which construct mechanisms which must meet certain conditions, dynamic and cinematic and pump-specific gearings, involves finding technologies of tooth finishes, assuring geometric and cinematic properties. The tooth-finishing method through copying presents special disadvantages because it introduces errors unacceptable for the cinematic conditions of the gearing. The construction of specialized machinery for processing teeth, which simulates the main movements of the generation of teeth , is also unacceptable, because of the complexity and huge variety of unevolventic profiles used in pump-specific gearings. As a result, the use and extension of these profiles' finishing through grinding is required, together with the entire series of advantages therefrom. It is true that the execution of grinders with generic profile involves some remarkable technological difficulties, but they are both solvable and economically justified.

Since the grinding as a process constitutes a gearing of two gears with their axes crossed the setting of the profile of the grinder's tooth is reduced to analyzing the gearage in compliance with the general conditions of grinding (the processing of the whole active profile of the gear, gearing continuity, the uniform distribution of splintering forces on the whole length of the gearing line, etc.).

In the case of a given gearing, the conjugated flanks of the teeth are given of two surfaces, one rolled and the other one wrapping, tangent in the contact point, which moves after the characteristic curve, in the case of monoparametrical wrapping.

To determine the areas together, the following algorithm problem is established:

-the law of motion of the gearing point in the immobile is chosen (related to the wheel axles in the gearing);

- the finding of the characteristic lines found in rotary coordinated systems, related to the two gears (the lines of contact) and the immobile system (the gearing line);

- it is taken into account that, at the time of contact, there must be no intersection of surfaces in the limits of portions used as active flanks of the teeth

- through the teeth contact lines are drawn surfaces which, in order to constitute the conjugated surfaces, must comply with the following conditions:

- a common normal to the contact surfaces, in all contact lines' points;

- to be perpendicular on the relative velocity vector's points and on the tangent to the lines of contact;

- the shape of joint surfaces and the position of the lines of contact must ensure compliance to the condition that the curvature of the area in the normal plane to the line of contact be at least equal to the geodhesic curvature of the contact line.

$$\frac{1}{\tau_g} \geq \frac{1}{2} \cdot \left(\frac{1}{\rho_1} \cdot \sin 2\varphi - \frac{1}{\rho_2} \cdot \sin 2\varphi \right) \quad (1)$$

$$\frac{1}{r} \cdot \cos\theta = \left(\frac{1}{\rho_1} \cdot \cos^2\varphi - \frac{1}{\rho_2} \cdot \sin^2\varphi \right) \quad (2)$$

In accordance with Figure 1, the notations have the following meanings:

$\frac{1}{\rho_1};\ \frac{1}{\rho_2}$ – *the main curvings of the conjugated surfaces*;

$\frac{1}{\tau_g}$ – *the geodhesic curving of the contact surface line*;

$\frac{1}{r}$ – *the curving of the contact line*;

θ – *the angle between the normal at the surface and the main normal to the contact line*;

φ – *the angle between the direction and the +angent to the curve having the main curving*

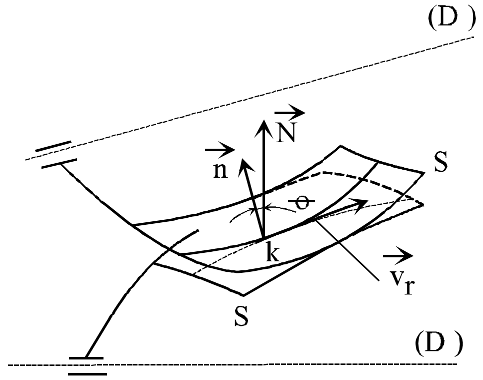

Figure 1. The elements of conjugated areas in the cross-axle grinding.

If we consider a contact point on the characteristic, provided that those two areas are not separating nor interpenetrating, is that the relative speed be perpendicular on the common normal.

$$\vec{v}_{r21} \cdot \vec{n} = 0 \quad (3)$$

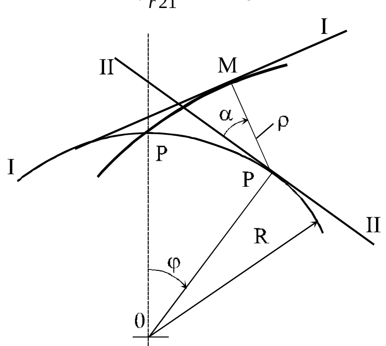

Figure 2. The elements of a tooth's profile.

In determining the actual profile of the grinder's tooth, in accordance with the general theory of conjugated profiles in frontal plane gearing, if we move on to the equivalent gearing, the problem is reduced to determining a profile conjugated to the profile given (the gear's profile equivalent to the processing wheel). In this regard it is considered a system of current polar coordinates (Figure 2). A current point M on the tooth's profile is defined through the normal PM =ρ to the profile and through the angle α it makes with the tangent in the point P. The arch P0P on the circle of radius Rr with the centre O is defined by the angle φ , which it makes with the position radius OP with the fixed radius OP0. For defining point M in the polar coordinate system, it is necessary that the defining elements are expressed based on a single variable(the angle φ). In this case, the equations of the point M of the profile are:

$$\rho = \rho(\varphi, R_r); \quad \alpha = \alpha(\varphi, R_r); \quad arcP_0P = \varphi \cdot R_r. \tag{4}$$

If radius Rr becomes infinite, the wheel turns into a cremailliere, and the current pole defining the profile moves on a right line called rolling line(figure 3). In this case, the segment $\overline{P_{oc}P} = x$ is variable, depending on which the equations of the current profile of the cremailliere are defined:

$$\rho_c = \rho_c(x); \quad \alpha_c = \alpha_c(x); \quad \overline{P_{oc}P} = x \tag{5}$$

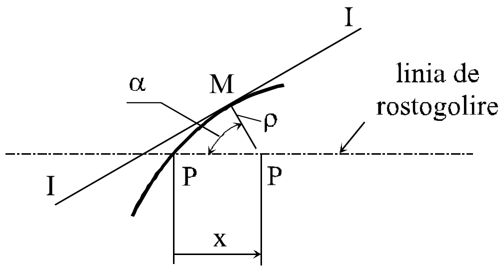

Figure 3. The rolling line of the cremailliere

Since the current point M belongs to both the profile of gear's 1 tooth(of radius Rr1 and angle φ1) and the profile of gear's 2 tooth(of radius Rr2 and angle φ2), the equations of definition in the two polar coordinate systems linked to the two gears are:

$$\begin{aligned} &\rho_1 = \rho(\varphi_1, R_{r1}); \quad \alpha_1 = \alpha(\varphi_1, R_{r1}); \quad arcP_{01}P = \varphi_1 \cdot R_{r1} \\ &\rho_2 = \rho(\varphi_2, R_{r2}); \quad \alpha_2 = \alpha(\varphi_2, R_{r2}); \quad arcP_{02}P = \varphi_2 \cdot R_{r2} \end{aligned} \tag{6}$$

It is noted that the equation of gear 2's tooth's profile conjugated to gear 1, is easily obtained from the equation of profile 1. Curve "a", representing the gearing line, being fixed (independent of the rotation of the gears) is defined in fixed polar coordinates with the fixed pole in P, whereas the reference line is represented by the common tangent II. The equations of the gearing line, in dependence with the rotation angle of gear 1, are:

$$\rho_a = \rho(\varphi_1, R_{r1}); \quad \alpha_a = \alpha(\varphi_1, R_{r1}) \tag{7}$$

Curve "c" that gears on both sides with gear 1 and gear 2, materializes in the profile of two cremaillieres, c1 and c2, one being the other's negative, and their profile equations are:

$$\begin{aligned} &\rho_{c1} = \rho(\varphi_1, R_{r1}); \quad \alpha_{c1} = \alpha(\varphi_1, R_{r1}); \quad \overline{P_{0c1}P} = x \\ &\rho_{c2} = \rho(\varphi_2, R_{r2}); \quad \alpha_{c2} = \alpha(\varphi_2, R_{r2}); \quad \overline{P_{0c2}P} = -x \end{aligned} \tag{8}$$

Since the rolling on the circles of radius Rr1 and Rr2 is done without sliding, the arcs on these circles are equal: The two profiles being conjugated with each other and with the cremailliere's profile, their equations may be expressed depending on x. Parametric equations, in polar coordinates, of the profile of the two cremaillieres, of the profiles of gear 1 and gear 2 and of the gearing line, depending on x are:

$$\rho_{c1} = \rho(x); \; \alpha_{c1} = \alpha(x); \; \overline{P_{0c1}P} = x \tag{9}$$

$$\rho_{c2} = \rho(x); \; \alpha_{c2} = \alpha(x); \; \overline{P_{0c2}P} = -x \tag{10}$$

$$\rho_1 = \rho(x);\ \alpha_1 = \alpha(x);\ arcP_1P = x \tag{11}$$

$$\rho_2 = \rho(x);\ \alpha_2 = \alpha(x);\ arcP_2P = -x \tag{12}$$

$$\rho_a = \rho(x);\ \alpha_a = \alpha(x) \tag{13}$$

Gear 1 was considered as equivalent to the processing wheel, and gear 2equivalent to the grinder. It is noted that, given that the profile of a gear is defined,the profile of the conjugated gear is defined immediately, as are the profiles of both the gearing line and the associated cremaillieres.

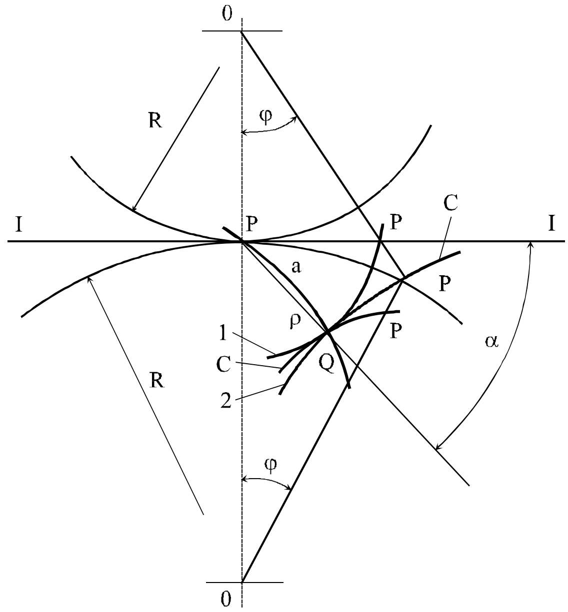

Figure 4. Elements of frontal gearing.

Another way of determining the profile of the tooth of the grinder that must process the teeth of a gear whose profile is given, and which leads to outstanding technological features, is based on the mathematical theory of rolling areas.

In the case of the cross-axled gearing, the complementary generating cremaillieres, with inclined teething, have a relative movement of such nature that their reference planes and flanks slide from one another. The reference planes perform translation movements, rolling without sliding on the rolling cylinders of the gears, thus forming two distinct gearings – the cylindrical wheel and the inclined-tooth cremailliere.

Figure 5 shows the mode of shifting from the gearing of two cross-axled gears, to the equivalent gearings – the cylindrical wheel and the inclined-tooth cremailliere.

Each cremailliere has a translation movement equal to the speed $v=\omega\cdot r$(where ω represents the angular speed, and r represents the radius of the rolling cylinder for the given gear), consequent to the slideless rolling over the rolling cylinders of the gears. For the gearing to take place, the profiles of the flanks of the gear's teeth must overlap in normal section, and the relative speed of the flanks in this section must be null. Since between directions of the speeds v1 and v2 there is an angle Σ, equal to the angle of crossing between the axles of the gears, there appears a relative sliding movement alongside the teeth, with the speed:

$$\vec{v}_{r21} = \vec{v}_2 - \vec{v}_1 \tag{14}$$

corresponding to the overlapped contact points of the two flanks.

Since the reference inclined-tooth cremaillieres (with the angle of inclination of the teeth equal to the angle of inclination of the gear to which the cremailliere corresponds) have their profiles overlapped in normal section on the direction of the tooth, the determination of the active profile of the conjugated gear(the grinder), - if the profile of one of the gears in the gearing is known(the processing wheel) – is accomplished following the algorithm:

- the profile of the cremailliere corresponding to the processing gear in normal section on its axis is determined;
- the profile of the cremailliere corresponding to the grinder in normal section on its axis is determined, profile which results from the profile of the cremailliere corresponding to the processing gear(the two cremaillieres have their profiles overlapped in normal section, in the direction of the tooth);
- the profile of the grinder is determined, from the profile of the corresponding cremailliere.

It is noted that, in this case, the profile of the conjugated gear (the grinder), is determined in normal section on its axis.

According to mathematical theory of wrapping surfaces, given a family of curves Cφ, dependant on the parameter φ, defined by the equation:

$$F(x, y, \varphi) = 0 \tag{15}$$

it admits a wrapper if there is a curve Γ that does not belong to that family and that verifies the following conditions:

- to each curve Cφ there corresoponds a point M on the curve Γ, and reciprocally, to each point on the curve Γ there corresponds a curve from the family;
- the curves Cφ and Γ are tangent in the point M;
- there are no common arcs between the curve Γ and the Cφ curves.

It results that the equation of the wrapping curve can be determined, solving the following system:

$$\begin{cases} F(x, y, \varphi) = 0 \\ F'\varphi(x, y, \varphi) = 0 \end{cases} \tag{16}$$

If we eliminate the parameter φ from this system, we obtain the equation of the curve Γ in implicit form, and if we cannot eliminate the parameter φ, we obtain the equation of the wrapping curve in parametric form. It is noted that, after developing the system, we can also obtain the coordinates of singular points of the curves Cφ.

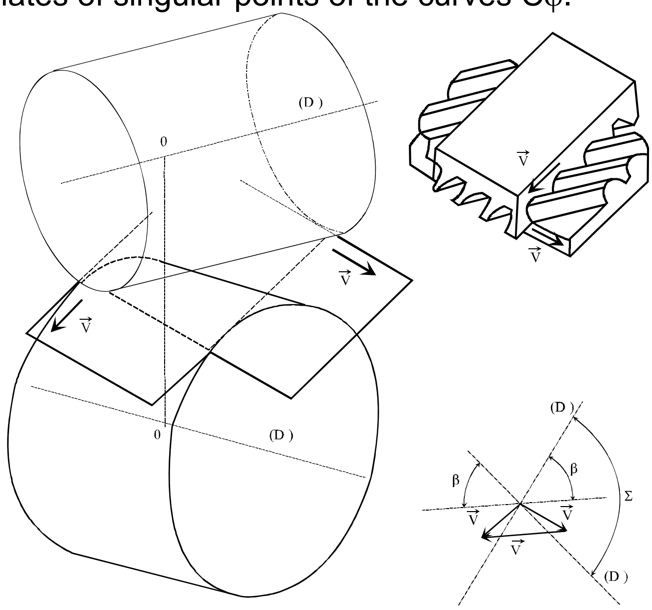

Figure 5. The gearing with cross-axles and the equivalent gearings, the cylindrical gear and the cremailliere

Following these general considerations, it is possible to determine the profiles for the cremailliere and the grinder. If there are considered two reference systems, xAy attached to the gear, and xPy attached to the cremailliere, linked by the independent parameter φ (which represents the angle of rotation of the gear), the profile C of the tooth of the gear in normal section on its axis being known, after moving to the reference system xPy, considering the slideless rolling of axis Px over the circle of radius r1, the equation of the curve family Cφ is obtained, in the reference system xPy, in the form of an equation of the type:

$$F(x, y, \varphi) = 0 \tag{17}$$

Through solving the system:

$$\begin{cases} F(x, y, \varphi) = 0 \\ F'\varphi(x, y, \varphi) = 0 \end{cases} \tag{18}$$

the equation of the curve γ is obtained, in the reference system xPy.

On the basis of the same ratiocination, if the equation of the cremailliere's profile in the system xPy is known,the equation of the profile of the gear in the reference system xAy is deduced.

In concordance with figure 6, the link between the coordinates of a certain point N in the two reference systems is deduced so:

$$\begin{cases} x_N = x_A + X_N \cdot \cos\varphi + Y_N \cdot \sin\varphi \\ y_N = y_A + Y_N \cdot \cos\varphi - X_N \cdot \sin\varphi \end{cases} (*) \tag{19}$$

reciprocally, it can be transposed as:

$$\begin{cases} X_N = (x_N - x_A) \cdot \cos\varphi - (y_N - y_A) \cdot \sin\varphi \\ Y_N = (x_N - x_A) \cdot \sin\varphi + (y_N - y_A) \cdot \cos\varphi \end{cases} (**) \tag{20}$$

represents the link between the coordinates of a certain point in two rotated and moved reference systems. xA and yA represent the coordinates of point A(the centre of the reference system xAy) in the reference system xPy. Since the circle of radius r rolls without sliding over the axis Px, the segment PP' is of length Γ·φ. The coordinates xA and yA are therefore given by the relations:

$$\begin{cases} x_A = \Gamma \cdot (\varphi - \sin\varphi) \\ y_A = \Gamma \cdot (1 - \cos\varphi) \end{cases} \tag{21}$$

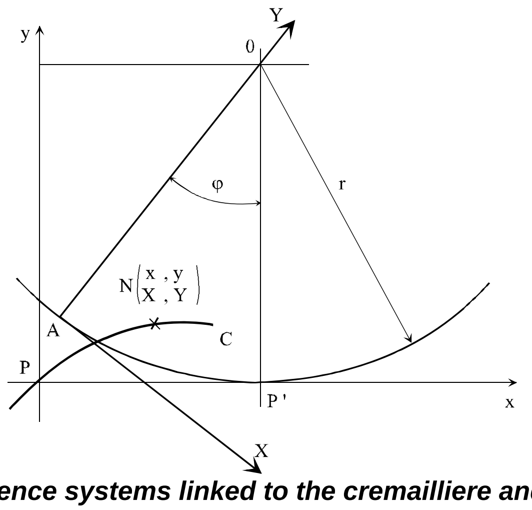

Figure 6. The reference systems linked to the cremailliere and to the gear

In conclusion, the coordinates of a certain point on the profile C of a gear's tooth or on the profile γ of the cremailliere, in the reference system linked to the cremailliere, and to the gear respectively, will be:

$$\begin{cases} x_c = r\cdot(\varphi-\sin\varphi)+X_c\cdot\cos\varphi+Y_c\cdot\sin\varphi \\ y_c = r\cdot(1-\cos\varphi)+Y_c\cdot\cos\varphi-X_c\cdot\sin\varphi \end{cases} \quad (22)$$

and:

$$\begin{aligned} X_\gamma &= \left[x_\gamma - r\cdot(\varphi-\sin\varphi)\right]\cdot\cos\varphi-\left[y_\gamma-r\cdot(1-\cos\varphi)\right]\cdot\sin\varphi \\ Y_\gamma &= \left[x_\gamma - r\cdot(\varphi-\sin\varphi)\right]\cdot\sin\varphi-\left[y_\gamma-r\cdot(1-\cos\varphi)\right]\cdot\cos\varphi \end{aligned} \quad (23)$$

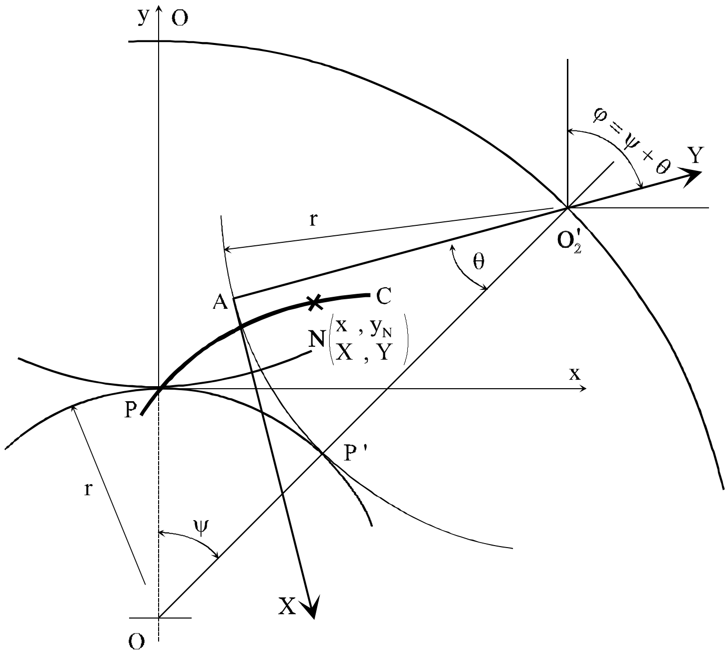

Figure 7. The reference systems linked to the gear and to the cremailliere

On the basis of this algorithm, the conjugated profile can easily be determined, afferent to the grinder, for the processing of the gear's profile. For the verification of the obtaining of a correct conjugated profile there can be done an analysis on the equivalent gearing. The study of the equivalent gearing allows, in its turn, the direct determining of the grinder's profile.

In the case of the analysis on the equivalent gearing, after establishing the equivalent rolling diameters, the establishing of the conjugated profiles can be done, with the same method as the previous case. To the wheel whose profile is known, the reference system xAy will be linked, whereas to the grinder, whose profile is yet to be determined, will be linked the reference system xPy. It is noted that the profiles being taken into account represent the profiles of the teeth in normal section on their direction. In concordance with figure 7 the link between the coordinates of a point N in the two reference systems can be determined.

In order to establish the link between the coordinates of the point N in the two reference systems the equations of the systems (*),(**) are used. The coordinates of the point A(xA, yA) are determined taking into account the rolling of the circles of radius r1 and rs. Between the angles of rotation ψ and ɵ there is the bond:

$$r_1\cdot\psi = r_s\cdot\theta;\quad \text{sau}\quad \theta=\frac{r_1}{r_s}\cdot\psi \quad (24)$$

From the figure there result the coordinates xA and yA of the point A:

$$x_A = (r_1 + r_s) \cdot \sin\psi - r_s \cdot \sin(\theta + \psi)$$
$$y_A = (r_1 + r_s) \cdot \cos\psi - r_1 - r_s \cdot \cos(\theta + \psi) \tag{25}$$

Therefore, the link between the coordinates of the point N in the two reference systems, depending on the angle of rotation of gear 1 will be:

$$x_N = (r_1 + r_s) \cdot \sin\psi + (Y_N - r_s) \cdot \sin\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right) + X_N \cdot \cos\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right)$$
$$y_N = (r_1 + r_s) \cdot \cos\psi + (Y_N - r_s) \cdot \cos\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right) - X_N \cdot \sin\left(\frac{r_1 + r_s}{r_s} \cdot \psi\right) \tag{26}$$

Given these considerations, it can be deduced that there has been established a unitary system for determining the profiles of cylindrical conjugated gears that gear with crossed axles. There can also be easily established the profile of the cremaillieres of the gears in gearing, which allows the establishing of the profile of the snail-mill for the processing through rolling of a given profile. The profile of the slotting machine's knife-gear's tooth can also be determined, for processing the channels. The profile of the tooth of the disc-grinder can be also determined.

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